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If $AC, AD, AE, AF, BC, BD, BE, BF$ are homographic, the six points A, B, C, D, E, F , no three of them being on a straight line, lie on a curve of the second degree. Thus, by the preceding proposition, the rays drawn from any two of the six points A, B, C, D, E, F to the other four are homographic systems.

Chasles proves these propositions in the inverse order to that here given. See *Traité de Géométrie Supérieure*, Chapter XXV, Arts. 544, 416.



SOLUTIONS OF EXERCISES.

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It is assumed that when a gate in a water-pipe is closing the pressure increases uniformly and the discharge decreases uniformly. Investigate an expression for the shortest safe time for closing the gate on the basis of these hypotheses: given the length of the pipe, the velocity of the stream, the working pressure, and the greatest admissible pressure to which the pipe may be exposed.

[*W. M. Thornton.*]

SOLUTION.

Put L for the length of the pipe,

V velocity of the stream,

Ω sectional area of the pipe,

g acceleration due to gravity,

D heaviness of water,

p working pressure in the pipe,

P highest admissible pressure in the pipe,

T required time in seconds.

The kinetic energy of the stream is

$$\frac{D}{2g} \cdot L\Omega V^2.$$

The velocity and pressure at t seconds after the closing of the valve begins are

$$V_t = V \left[1 - \frac{t}{T} \right], \quad p_t = p + (P - p) \frac{t}{T};$$

and the work done by the stream in entering this region of high pressure is

$$\begin{aligned}\Omega \int_0^T P_t V_t dt &= \Omega V \int_0^T \left(1 - \frac{t}{T}\right) \left[p + (P - p) \frac{t}{T}\right] dt \\ &= \frac{1}{6} \Omega V (P + 2p) T.\end{aligned}$$

If we neglect the work done against pipe friction we may equate the kinetic energy to the work done. We get thus for the required time

$$T = \frac{D}{g} \cdot \frac{3LV}{P + 2p};$$

and conversely for the water-ram produced by closing the gate in T seconds

$$P - p = \frac{D}{g} \cdot \frac{3LV}{T} - 3p.$$

[*Wm. M. Thornton.*]

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SHOW geometrically (without using the calculus) that the asymptote of the hyperbolic spiral, $r = a : \theta$, is parallel to the initial line and distant a above it.

[*L. G. Barbour.*]

SOLUTION.

The Cartesian co-ordinates of a point on the spiral are

$$x = a \frac{\cos \theta}{\theta}, \quad y = a \frac{\sin \theta}{\theta}.$$

When θ decreases without limit x increases without limit, but y converges to limit a . Hence $y = a$ is asymptotic to the spiral. [*L. G. Barbour.*]

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DETERMINE the maximum right cone B inscribed in a given right cone A , the vertex of B being at the centre of the base of A . [*O. Root, Jr.*]

SOLUTION.

If the altitude of A be a and that of B be $a - x$, the area a of the base of B will be proportional to x^2 , and its volume to $y = x^2(a - x)$; and for the maximum

$$\frac{dy}{dx} = 2ax - 3x^2 = 0;$$

$$\therefore x = \frac{2}{3}a, \text{ and } a - x = \frac{1}{3}a.$$

[*T. U. Taylor.*]

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GIVEN the perpendicular, median, and bisector issuing from one and the same vertex of a plane triangle and terminating in the opposite side, to construct the triangle and determine a formula for its area. [Marcus Baker.]

SOLUTION.

1. *Construction of the triangle* This part of the problem is old.

2. *Calculation of the area.* Let p, q, r be the given perpendicular, bisector, and median, and P, Q, R their points of termination. Put l, m, n for QR, RP, PQ respectively and $2x$ for the base. Then the area is px and x is determined from the condition [Enc. VI, 3]

$$\frac{x+l}{x-l} = \frac{\sqrt{p^2 + (x+m)^2}}{\sqrt{p^2 + (x-m)^2}},$$

whence

$$x^2 = \frac{lr^2 - ml^2}{n};$$

$$\therefore J = p \sqrt{\frac{lr^2 - ml^2}{n}}.$$

Or, as will be found after obvious reductions,

$$J = p \sqrt{\left[(2p^2 - q^2) \sqrt{\frac{r^2 - p^2}{q^2 - p^2}} - (2p^2 - r^2) \right]}.$$

[W. M. Thornton.]

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GIVEN a pair of points A, B , if C, D are such that $OD \cdot OC = OA^2 = OB^2$ and $AOC = AOD$, prove that the pair A, B bears similar relation to the pair C, D . Show, also, the existence of a pair E, F which bears the same relation to each of the pairs A, B and C, D . [Wm. Woolsey Johnson.]

SOLUTION.

1. Assume O to be the middle point of AB . As an easy deduction from the conditions given, the triangles COA and AOD are similar; also the triangles COB and BOD .

$$\angle CAD = \angle CAO + \angle OCA = \angle COB = \angle BOD$$

$$\text{and} \quad \angle CBD = \angle CBO + \angle OCB = \angle COA = \angle AOD;$$

$$\text{also} \quad (OB =) AO : CB :: OD : BD$$

$$\text{and} \quad (OA =) BO : CA :: OD : AD;$$

\therefore the triangles AOD and CBD are similar;

also the triangles BOD and CAD are similar.

If P be a point in CD such that $\angle BAD = \angle CAP$, since $\angle OBD = \angle ACD$, the triangles BAD and CAP are similar.

Dividing $OB (= AO) : OD :: AC : AD :: CB : BD$

by $AP : AD :: CP : BD$,

we have $AC : AP :: CB : CP$;

and, since $\angle OAD = \angle CAP = \angle BCD$, the triangles CAP and BCP are similar, and

$$AP : CP :: CP : BP, \text{ or } AP \times BP = CP^2;$$

similarly it may be shown that $AP \times BP = DP^2$.

2. Let $C'A$ be a line parallel to BD through A , and Q a point such that $\angle BQC = \angle C'AC$ and $BQ : CQ :: BD : CA$; then the triangles BDQ and CAQ will be similar and also the triangles CQB and AQD .

Let FQ be a line bisecting $\angle AQB$ and put $FQ^2 = AQ \times BQ$; continue FQ to the point E such that $QE = FQ$; then will E and F satisfy the conditions required. [Ormond Stone.]

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IN THE TRIANGLE ABC draw AD to the point D in BC ; then will

$$AB^2 \cdot DC + AC^2 \cdot DB = BC \cdot AD^2 + BD \cdot DC \cdot BC.$$

[J. R. Spiegel.]

SOLUTION.

From C and B draw CE and BF meeting AD at right angles at E and F . Since ED and DF are the projections of BD and DC respectively on AD , we have

$$AB^2 \cdot DC = AD^2 \cdot DC + BD^2 \cdot DC + 2AD \cdot ED \cdot DC,$$

$$AC^2 \cdot BD = AD^2 \cdot BD + CD^2 \cdot BD - 2AD \cdot DF \cdot BD.$$

Adding, we obtain the required relation, since $DC + BD = BC$, and, by similar triangles, $ED \cdot DC = DF \cdot BD$. [A. V. Lane.]